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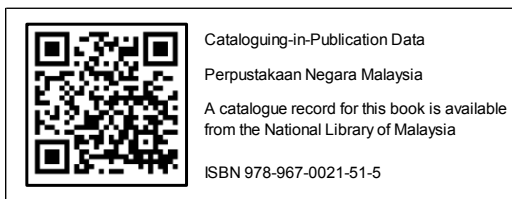
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Science  
Foundation  
**MATHEMATICS II**

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## PREFACE

This book covers the full syllabus of the Mathematics II course for Science Foundation Program students at Universiti Malaysia Kelantan. It has been written as a hands-on workbook module style to prepare these students in transitions from school mathematics teaching and learning concepts to higher mathematical thinking with analytical rigor at the university level.

In order to be user-friendly and easy to read and understand, the material was written casually. There is not contains a lot of text, and exercises, as in traditional textbooks. It is all mixed together with explanation, exploration, examples, exercises and tutorial worksheet for each chapter. It was designed for students to study in a small collaborative group setting, where they read and work together, assisting one another in mastering the content. Ideally, by using this book the students itself can be an instructor to their friends to clarify ideas. Although the intention of this book is to be used in solving some exercises in class, most of the reading and work will have to be done by the students out of class.

As previously indicated, it is preferred if students go through this book and work through the exercises in a small group setting both in and out of class. Because each exercise set can take many hours to complete, it is not always possible for group members to finish all of them at the same time. As a result, while certain tasks need the student to debate concepts with others in a group, they can also work on the exercises by themselves alone. If the student is extremely driven and has a strong background in mathematics, the content of this book can also be learned in a self-paced or individual manner. Ideally, all of the work should be done and shown in this workbook.

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**Siti Aisyah Nawawi**





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Our present and future students will really understand us through this book. Therefore, we would like to express our gratitude to previous students which provided a space and time for us to write and gather materials through teaching and learning experiences with them for mathematics and statistics subjects to such an extent this book can be realized. We also want to give our special thanks to many people who saw us through this book; to all those who provided support, talked things over, read, wrote and offered comments.

We also would like to thank our family for enabling us to publish this book. They supported and encouraged us in spite of all time it took us away from them.

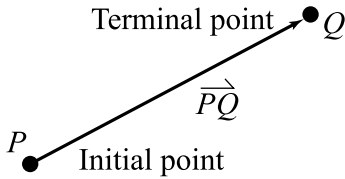
Thanks to UMK Science Preparatory Department and staff members of UMK Jeli Campus for encouraging us to write this book. Special thanks also goes to University Malaysia Kelantan for the one book one subject (OBOS) campaign that has further encouraged and motivated us to write this book. We really hope that this book will benefits to Science Foundation Program students in preparing them to strive in mathematics and statistics courses during undergraduate program.



# CHAPTER 1

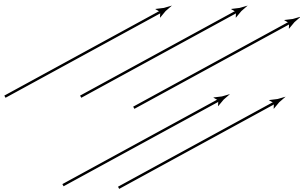
## VECTORS

### 1.1 DEFINITION OF VECTORS



Vectors are usually first introduced as objects having magnitude and direction such as force and velocity which cannot be completely characterized by a single real number. Geometrically,

such a quantity can be represented by using a directed line segment. The directed line segment has initial point  $P$  (tail of  $\vec{PQ}$ ) and terminal point  $Q$  (head of  $\vec{PQ}$ ). Its magnitude (or length) is denoted by  $\|\vec{PQ}\|$  and can be found using the Distance Formula.

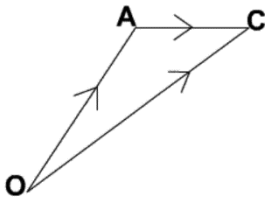


Two directed line segments that have the same magnitude and direction are Equivalent as illustrated by side figure. The set of all directed line segments that are equivalent to the directed line segment  $\vec{PQ}$  which can be

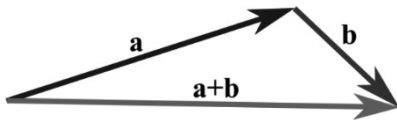
said as a vector in the plane. Vectors defined this way are called free vectors.

Therefore, any two vectors of the same length and parallel to each other are considered to be identical. So, by this definition a vector is an infinite set of parallel directed line segments. Vectors can be denoted lowercase, boldface letters such as  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  or lowercase with arrows such as  $\vec{u}$ ,  $\vec{v}$ ,

and  $\vec{w}$ . Therefore, for above case the vector  $\overrightarrow{PQ}$  can be denoted as  $\mathbf{v} = \overrightarrow{PQ}$  or  $\vec{v} = \overrightarrow{PQ}$ , where its magnitude is also can be denoted as  $\|\mathbf{v}\|$  or  $\|\vec{v}\|$  instead of  $\|\overrightarrow{PQ}\|$ .



People often choose one line segment from this infinite set to suit a particular application and it is sensible to ask why in practice we can take a single representative without reference to the whole set. For example, suppose an insect walk directly from a point O to point A and then from point A to point C with the length 30cm and 20cm respectively. The insect has walked 50 cm altogether, but in clearly not 50 cm from O to C,  $A + AC \neq OC$ . However it is true to say that, in traveling from O to A and then from O to C, or travelling from O to A and A to C, the insect arrives at the same point C as same as the insect walk directly from O to C,  $\overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}$ , where  $\overrightarrow{OA}$  indicates both, the length and direction that are being considered.



By working with just the geometric definition of the magnitude and direction of vectors, several operations on vectors such as addition, subtraction, and multiplication by scalars with their properties can be defined. For addition of vectors, Given two vectors  $\mathbf{a}$  and  $\mathbf{b}$ , a sum of these vectors can be denoted as  $\mathbf{a+b}$ , and can be defined as follows. Vector  $\mathbf{b}$  is translated until its tail coincides with the head of  $\mathbf{a}$ . Then, the directed line segment from the tail of  $\mathbf{a}$  to the head of  $\mathbf{b}$  is the vector  $\mathbf{a+b}$ .